## CRAIG, KELLER, AND HAMMEL

with  $\Phi$  and obtain using (40)

$$\Gamma = \frac{12(\rho_n/\rho) v_n^2 \eta_n}{sTd^2}$$
(43)

Upon integrating  $\Gamma$  along the length of the slit, the total normal fluid flux change within the slit is found:

$$\int_{T_0}^{T_1} \Gamma \, dz = \int_{T_0}^{T_1} \Gamma \, \frac{dz}{dT} \, dT = -\int_{T_0}^{T_1} \frac{\bar{\mathbf{q}}^2}{T^2 s_\lambda \Lambda d^2} \frac{\Lambda d^2}{\bar{\mathbf{q}}} \, dT$$

$$= -\frac{\bar{\mathbf{q}}}{s_\lambda} \int_{T_0}^{T_1} \frac{dT}{T^2} = \frac{\bar{\mathbf{q}}}{s_\lambda} \left(\frac{1}{T_1} - \frac{1}{T_0}\right).$$
(44)

However, at any temperature T,  $\bar{q} = \rho s T \bar{v}_n$ , so that (44) may be written

$$\int_{T_0}^{T_1} \Gamma \, dz = \left(\frac{\rho s T \bar{\mathbf{v}}_n}{s_\lambda T}\right)_1 - \left(\frac{\rho s T \bar{\mathbf{v}}_n}{s_\lambda T}\right)_0 = (\rho_n \, \bar{\mathbf{v}}_n)_1 - (\rho_n \, \bar{\mathbf{v}}_n)_0 = \overline{\Delta \mathbf{N}}, \quad (45)$$

the change in normal fluid flux per unit area over the length of the slit.

The role of  $\Gamma$  in the two fluid equations of motion has been discussed by Zilsel (30), and we may readily show using his equations that the perturbation introduced by the dissipation term is exceedingly small. Thus the Rayleigh dissipation term does not contribute to the heat flux (where its effect would be appreciable) but rather the dissipation present generates normal fluid without appreciably influencing the overall heat transport or temperature gradient.

The term analogous to the Rayleigh term for the Gorter-Mellink force is

$$\Phi_{\rm GM} = A \rho_{\rm s} \rho_{\rm n} \left( \left| \mathbf{v}_{\rm s} - \mathbf{v}_{\rm n} \right| - \mathbf{v}_{\rm c} \right)^2 (\mathbf{v}_{\rm s} - \mathbf{v}_{\rm n})^2.$$
(46)

When this term is added to the Rayleigh term the total change in momentum flux can be calculated in a manner similar to that used in Eq. (44). The only change introduced occurs in the expression for dT/dz for which Eq. (25) is used. The final result (45) is unchanged, for the terms involving the Gorter-Mellink coefficient A drop out. It is likely that there is no dissipation associated with the Gorter-Mellink term (31).

## III. NUMERICAL SOLUTION OF FLOW EQUATIONS

To solve the nonlinear integral equation (26) use is made of the fact that for given  $T_0$  and  $T_1$  the average heat current density  $\bar{\mathbf{q}}$  through the slit is a constant. Therefore with a fixed  $T_0$  chosen as well as a particular value for  $\bar{\mathbf{q}}$  Eq. (26) is integrated numerically out to such a  $T_1$  that equality is obtained. The heat current  $\bar{\mathbf{q}}$  is then increased by a small increment, and a new (larger) value for  $T_1$  computed. This procedure is repeated until  $T_1$  reaches  $T_{\lambda}$ . Thus the entire  $\bar{\mathbf{q}}$ ,  $T_1$  curve is obtained. A new value for  $T_0$  is then selected and the entire process repeated. In this way the family of heat flow curves is generated.

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